PROBLEM SET OF VISITING SOPHIA

SOCRATES

Χαῖρε, friend. It pains me to admit it, but after all this tail-chasing brought on by those idiot technocrats, I am in need of a break! I am sure you could use one, too.

Thankfully, I have plans for the week! I will be visiting my sister Adelphia and niece Sophia! They live just a quick flight away in Bubo, the capital of Pallassa. I love visiting Pallassa, since it has so many wonderful universities. Well, at least East Pallassa does. West Pallassa has always been a bit behind, perhaps because of all the people who join the Medusaheads—that's a nickname for the Pallassan elite guard. But you already knew that.

I haven't seen Sophia since she was 7, and now she nearing age 10. I spend too much time in my stupid office on that stupid half-island. I am so excited to see her...but, I don't quite know what to get her as a sorry-I've-been-such-a-bad-uncle present. I've been reading some blogs about what books to buy for precocious girls like her, but I'm still on the fence. Perhaps you can help me?

Anyway, it will be good to rest and think. I hope you will be able to do the same for the week. Until then: *χαῖρε!*

This is so embarrassing. All of the blogs I read told me that I should get Sophia one of the books from the *Ioanna Patsasoglou* series. But there are so many of them! The blogs all said that some of them were really good, some were good, some were just OK, some were bad, and some were really bad. But none of the blogs say the same thing about them!

Of course, as I am a good scholar, I have compiled data prior to making my purchase. Surely Sophia will be impressed by my efforts. What's that you say? Kids don't care how hard you work to get a present? Balderdash. Though you may be on to something...

Anyway, here are the data I compiled about the books. Each entry is the proportion of blogs that said a given book (rows) is of a given quality (columns). I think we should give each kind of book a score: really good books get a score of 2; good books get a score of 1; meh books get a score of 0; bad books get a score of −1; and really bad books get a score of −2.

Oh! I need to get to my flight! Can you do the calculations and text me the results so I can buy the right book at the airport? kthxbai

PART 1 | WHICH BOOK?

BOOK	VERY GOOD	GOOD	MEH	BAD	VERY BAD
Ioanna Patsasoglou and the NECROMANCER'S CHOLELITH	0.0	0.0	0.2	0.4	0.4
Ioanna Patsasoglou and the ENCLAVE OF KASSANDRA	0.1	0.0	0.0	0.0	0.9
Ioanna Patsasoglou and the CONVICTED CYCLOPS	0.0	0.5	0.3	0.1	0.1
Ioanna Patsasoglou and the INSULATED DISKOPOTIRO	0.3	0.0	0.0	0.6	0.1
Ioanna Patsasoglou and the BENNU TAXIS	0.2	0.3	0.5	0.0	0.0
Ioanna Patsasoglou and the HIRSUTE COUSIN	0.1	0.0	0.0	0.5	0.4
Ioanna Patsasoglou and the PRAYER THAT BACKFIRED	0.1	0.4	0.4	0.1	0.0

This shouldn't be too hard, but that's easy for me to say.

1. Using the utility scores mentioned on the previous page, compute the expected utility for each of the seven books.
2. Using the expected utilities you computed, draw a preference doodle for the books, where there is an arrow from one book to another book if and only if the first book has an expected utility at least as high as the second.
3. Which of the books are better (in expectation) than a book that's Meh for sure?
4. Suppose now that I wanted to avoid looking like an idiot, so that now a Very Bad rating yields −5 happiness points instead of −2. Does the ranking change?

necessary for **PASS**: get 3

sufficient for one **ALMA**: get 4

PART 2 | PREFLIGHT COFFEE

I stopped at Aglaopes for some coffee in the terminal. A strange thing occurred to me as I was adding some sugar to the cup: if I add one grain of sugar to my cup of coffee, it is an entirely new thing than it was before! And yet, my taste buds cannot detect the difference—I am not that fancy. Similarly, I cannot detect differences between one grain and two grains, between two grains and three grains, three and four, ..., one million and one million plus one, and so on. And yet, I can definitely tell the difference between black coffee and a cup of coffee with a million grains of sugar in it, and I most definitely prefer the former!

Is this a problem?

1. Suppose I wanted to state the following: "the indifference relation $\sim$ is transitive." What would that look like in symbols?
2. Assuming the indifference relation $\sim$ is transitive, is the following TRUE or FALSE: if w is indifferent to x, x is indifferent to y, and y is indifferent to z, then w is indifferent to z. In other words, can you concatenate transitivity?
3. Is the following TRUE or FALSE: if the weak preference relation $\succsim$ is transitive, then the indifference relation $\sim$ is also transitive.
4. In light of what you've just learned, what "should" I prefer between a cup of coffee with no sugar and one with enough sugar to fill the known universe?

necessary for **PASS**: get 2

sufficient for one **ALMA**: get 4

sufficient for another **ALMA**: list three kinds of decision problems where sensory limitations might factor in, and three other kinds of decision problems where sensory limitations might not matter.

PART 3 | MY AIRPLANE DREAM

Once I got onto the plane, I found myself asleep. Being old has its benefits! I think all that sugar must have gotten to me, because I had a really vivid dream. Let me tell you about it.

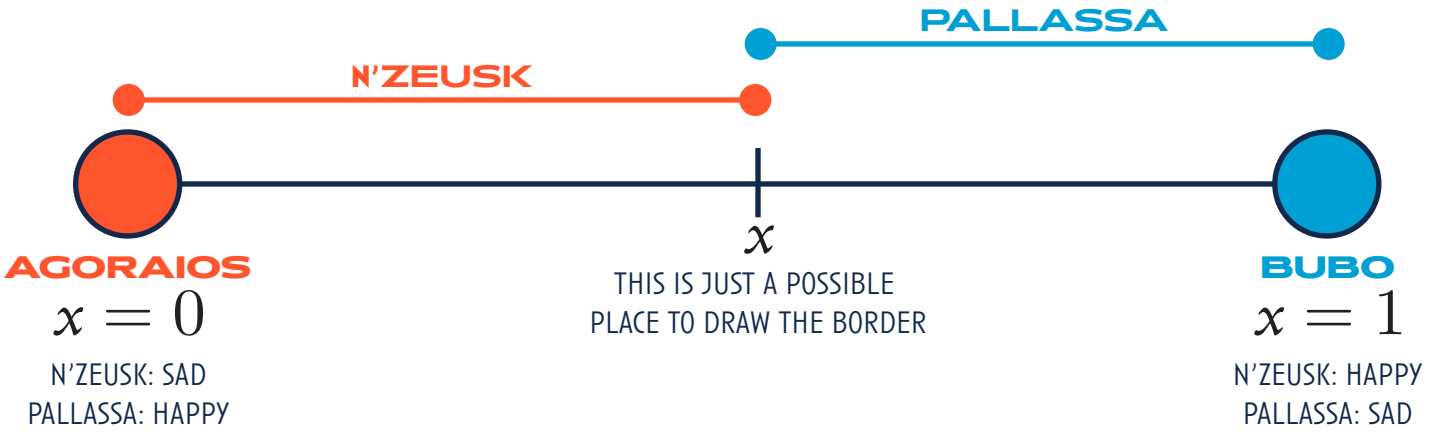
Soon after Adelphia and Sophia moved to Pallassa, there was a crisis between Pallassa and its powerful northern neighbor, n'Zeusk. The Zeuskateers (not the real demonym, but I can't help being a bit derogatory about them) wanted to redraw the border to take over some of West Pallassa, but the headstrong generals in Promachos and admirals in Polios and Poliouchos would have none of it. I was so scared! Things like that didn't happen back home in Lemnos. After a few Μπουμπούκι too many on the plane, I passed out and had one of those Plane Dreams about the whole incident. And it was in our new language!

Let us say that the Zeuskians and Pallassans were deciding how much of the territory each should get. In particular, the set of peaceful outcomes is

$$X = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\},$$

so that any x is what proportion of the n'Zeusk has, while $1 - x$ is the remaining proportion of land for Pallassa. So, I think maybe we should give peaceful-outcome scores like $u_Z(x) = x$ and $u_P(x) = 1 - x$.

As for war: if the crisis hadn't been stopped before war broke out, then the Zeuskians would have won with probability p and the Pallassans would have won with probability $1 - p$. (You can only win or lose against n'Zeusk; they are not known for compromise.) Regardless, fighting would have cost each state c_Z and c_P, respectively. I think winning is probably worth 1 point and losing is probably worth 0 points.



THE PEACE PROCESS

In my dream, I was asking *myself* questions. I think that is because I never quite understood why the Zeuskians and Pallassans managed to make peace—I was so sure that war was about to break out and that Adelphia and Sophia would be in big trouble. Here are some of the things I was wondering about:

1. If things are all laid out as I just described, then what is n'Zeusk's expected utility of fighting?
2. Which peaceful deals does n'Zeusk weakly prefer to fighting the war?
3. How about Pallassa? What is their expected utility of fighting?
4. And which peaceful deals does Pallassa weakly prefer to fighting the war?
5. Which peaceful deals did both states prefer to fighting the war? I wonder if this tells me something useful....
6. Draw a picture like the one I drew above, but where you highlight the peaceful deals both states prefer to fighting. Label the endpoints appropriately, both conceptually and mathematically! I will lose track otherwise!

necessary for **PASS**: get four

sufficient for one **ALMA**: get six

sufficient for a second **ALMA**: for both *p* and *c*, hypothosize three real-world varianbles that could plausibly influence the theoretical concept in question (*i.e.*, what are real-world things that influence *p*? how about *c*?)