PROBLEM SET 05

THE ARCHAIC TEXT

GENESEO

SOCRATES

Χαῖρε, friend. I've been in the library to pore over some old tomes on notions of fairness and goodness and all that is just. Goodness gracious, there are a lot of books about that. I've written none of them, but I still see myself quoted now and again. It's nice to have a fan, but I still feel a bit apologetic.

At any rate, during my studies I came across a passage that spoke to me—nay, *sung* to me—nay, *haunted* me—nay, *slayed* me—nay, *cut me to the quick*—nay, *shook my core*—nay, *rocked my foundations*—nay, *killed me until I died*. But not really.

The passage is a bit philosophical, but as it happens I'm a lover of wisdom. [*And robots, assuming they're sufficiently perfect.*] I was trying to find a problem to apply the ideas to, and then lo and behold, I turned on the news and learned about some bickering going on in the Heranian Tethysium. I think that we can use this controversy to learn about the old philosopher's ideas, but only if we set the problem up properly. My friend, once again the Hour of Rumination is upon us. *We begin!*

THE TETHYSIUM

PAGÓNIA

AGELÁDES

The Heranian Tethysium is dominated by two political parties: the **Peacocks** and the **Cows**. The libertine, urbane **Peacocks** prefer progressive policies on cultural questions and foreign affairs, whereas the traditionalist, austere **Cows** are more conservative on these issues. At the moment, both of these sets of issues square on relations with neighboring Hadestum.

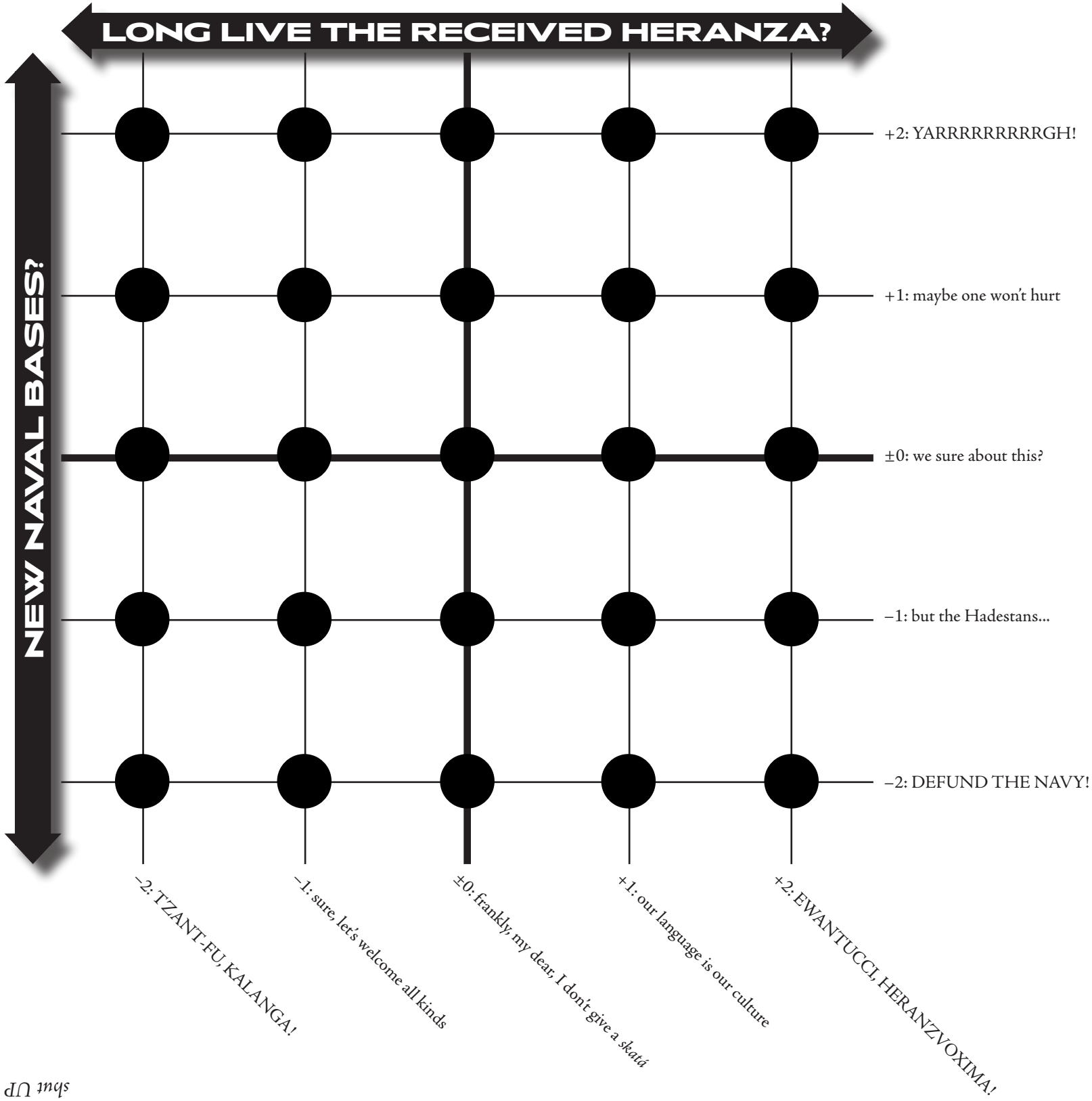
Culturally, the Heranians are trying to figure out whether to keep the Received Heranza as the official state language or to accommodate other languages; among these are the Vulgar Rhean spoken by indigenous peoples, the Neoclassical Hadestii spoken by Hadestan migrant workers looking for a new life working the canal in New Rodi, and the Persephe-Hadestii spoken by traders from Theristiki Mana. Generally speaking, the **Peacocks** favor linguistic pluralism, and the **Cows** favor maintaining the Received Heranza status quo.

Militarily, the Heranians are trying to figure out whether to place new naval bases in Lakes Heracles and Hippolyta, all the better to lowkey flex at Hadestum. Again, the increased industrial output in New Rodi makes for a far easier munitions build-up than back in the pre-canal days—which is to say, back in the days when the Hadestans routinely kicked the Heranians' *vlakas* in border skirmishes. Generally speaking, the **Peacocks** prefer maintaining the status quo, and the **Cows** favor beefing up the naval bases.

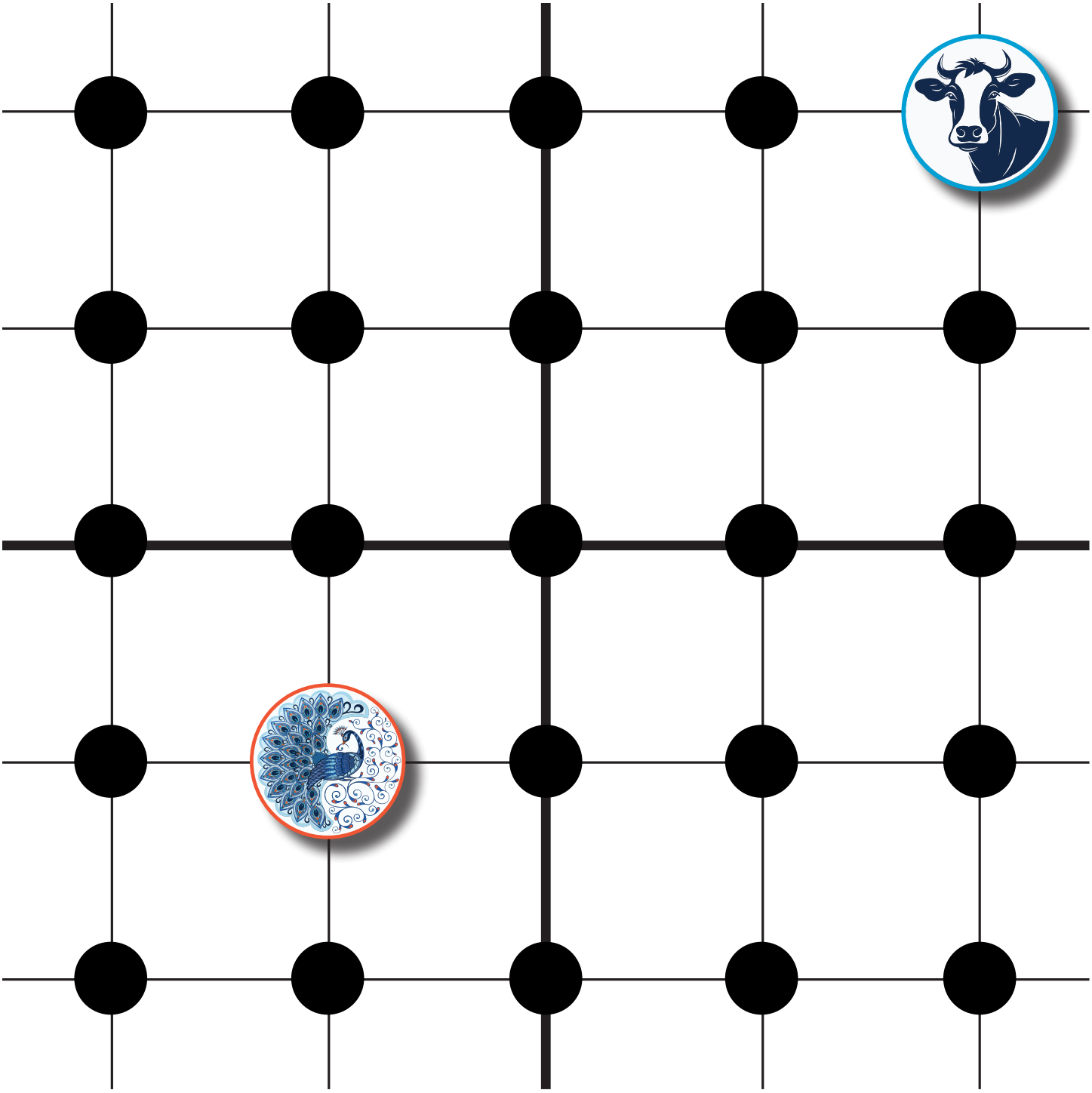
Finally, it seems as if the **Cows** have a more unified front on both of these issues, but will that wind up mattering?

PART 1 | RESPECT MY SPACE

I saw some polls where people could report their preferences on each issue along a five-point scale; one could report a preference that's strongly progressive, moderately progressive, neutral, moderately traditionalist, or strongly traditionalist. Working from left to right, we can assign these integer values ranging from −2 (for strongly progressive) to 2 (for strongly traditionalist), with neutral falling right in at 0.



shut UP



Let us agree to call the linguistic dimension Issue 1 and the militarization dimension Issue 2; I will represent these with superscripts.

The two parties have *ideal points:* favorite policies such that policies closer to the ideal point yield higher utilities. The Peacocks' ideal point is $\widehat{x}_P = (-1, -1)$, whereas the Cows' ideal point is $\widehat{x}_C = (+2, +2)$. Given these ideal points, the parties' utilities over policies take a particularly simple form:

$$u_P\left(x^1, x^2\right) = -\left|x^1 - \widehat{x}_P^1\right| - \left|x^2 - \widehat{x}_P^2\right|, \text{ and}$$
$$u_C\left(x^1, x^2\right) = -\left|x^1 - \widehat{x}_C^1\right| - \left|x^2 - \widehat{x}_C^2\right|.$$

So now, let's think:

1. What is the set of possible policies? You might use roster or set-builder notation, or you might take advantage of an operation we discovered before....
2. Write out each party's utility more explicitly by substituting in the relevant ideal point coordinates.
3. What is a party's utility at their ideal point? How do you know this maximizes their utility?
4. What are the parties' utilities at the neutral-neutral policy?
5. Is the policy $(-2, +2)$ Pareto-dominated? If so, by what?
6. Place the policies into a utility imputation space, taking note of which policies are assigned to which points in that space. (Some imputations might arise from multiple policies, so be careful!)
7. What is the set of Pareto-optimal *policies*? Again, you might use roster notation, set-builder notation, an operation, or even a drawing this time.
8. Suppose that the parties must make a deal or else a status quo policy of $(+1, -2)$ obtains. What is the bargaining range?
9. Suppose the status quo policy was instead $(-2, +2)$. What's the bargaining range now?
10. We say a party has a *first-best policy* if and only if there exists a unique policy that yields a strictly-higher utility than any other policy. I think that means that any time a party has a first-best policy, that policy must be Pareto-optimal. Do you think I'm right or wrong? Why or why not?

necessary for **PASS**: get 6

sufficient for one **ALMA**: get 10

sufficient for another **ALMA**: given a set X, we say a function $d : X \times X \to \mathbb{R}$ is a *distance* function if and only if it satisfies the following:

1. for all $x \in X$, $d(x, x) = 0$;
2. if $x \neq y$, then $d(x, y) > 0$;
3. for all pairs of alternatives x and y, we have $d(x, y) = d(y, x)$; and
4. for all triplets of alternatives x, y, and z, we have $d(x, z) \leq d(x, y) + d(y, z)$

Determine whether the parties' utilities are the negative of a distance function from the ideal point.

do you think these axioms of distance make sense for, say, the physical reality we find ourselves in?

imagine that I wanted to measure walking distance by calories burned, or driving distance by gasoline burned. would any of these four axioms of distance seem out of place?

Anyway, on to the old book I found. Here's the passage that pierced my soul:

> "The age we live in is a busy age; in which knowledge is rapidly advancing towards perfection. In the natural world, in particular, every thing teems with discovery and with improvement. The most distant and recondite regions of the earth traversed and explored the all-vivifying and subtle element of the air so recently analyzed and made known to striking evidences, were all others wanting, of this pleasing truth.
>
> "Correspondent to discovery and improvement in the natural world, is reformation in the moral; if that which seems a common notion be, indeed, a true one, that in the moral world there no longer remains any matter for discovery. Perhaps, however, this may not be the case: perhaps among such observations as would be best calculated to serve as grounds for reformation, are some which, being observations of matters of fact hitherto either incompletely noticed, or not at all would, when produced, appear capable of bearing the name of discoveries: with so little method and precision have the consequences of this fundamental axiom, ***it is the greatest happiness of the greatest number that is the measure of right and wrong***, been as yet developped."

Woweewowow! How could I possibly encode such wisdom? Well, let us suppose that the proportion of Heranian citizens that support the Peacocks is $\alpha \in [0, 1]$ and that the proportion of them that support the Cows is $1 - \alpha$. Now I can conceive of the total welfare in Herania at a policy x using the formula

$$W\left(x^1, x^2, \alpha\right) = \alpha u_P\left(x^1, x^2\right) + (1 - \alpha) u_C\left(x^1, x^2\right).$$

This formula assigns each policy a number representing total welfare—in other words, it helps us find the policy that generates the greatest good for the greatest number!

I read in the news that the Peacocks' preferences have changed a bit; they are now twice as sensitive on foreign-policy distance as they are on language-policy distance, making the utilities

$$u_P\left(x^1, x^2\right) = -\left|x^1 - \tilde{x}_P^1\right| - 2\left|x^2 - \tilde{x}_P^2\right|, \text{ and}$$
$$u_C\left(x^1, x^2\right) = -\left|x^1 - \tilde{x}_C^1\right| - \left|x^2 - \tilde{x}_C^2\right|.$$

Let's use these new utilities and the new welfare function to see if the old philosopher had a deep insight for we mortals.

You know how we do:

1. Using these updated utility functions, what is the set of Pareto-optimal policies?
2. What is the welfare score for the neutral-neutral policy when $\alpha = 1/2$?
3. What is the welfare for policy $(+2, +1)$ when $\alpha = 7/8$?
4. Which policy(ies) maximizes welfare when $\alpha = 0$?
5. Which policy(ies) maximizes welfare when $\alpha = 1$?
6. Which policy(ies) maximizes welfare when $\alpha = 1/2$?
7. Is it possible for a Pareto-dominated policy to maximize societal welfare for any α? Why or why not?
8. Is it TRUE or FALSE that any Pareto-optimal policy maximizes societal welfare at some α? Why or why not?
9. Evaluate the claim: a policy is Pareto-optimal if and only if it maximizes societal welfare for some α.

necessary for **PASS**: get 4

sufficient for one **ALMA**: get 9

sufficient for another **ALMA**: name the philosopher and the source

provocative question: does α really need to reflect the proportions of society? what other factors might influence the α a group is assigned—that is, the amount they matter for our conception of societal welfare? be as cynical as you'd like. just ruminate.