PROBLEM SET 06

VOLO CONTENDERE!

THE HERMESIATICS

SOCRATES

Χαίρε, friend. I feel like I've had a bit of a eureka moment lately; it's occurred to me that many of the goings-on around here are influenced not just by the decisions of prominent Olympians, but also by the way their decisions relate to one another. This truism is obvious to anybody that's ever visited the Hermesiatics, where the laws are few and the traders many.

The Hermestes value competition above nearly everything else, and it pervades most aspects of their culture. More often than not these competitions are fair and profitable, though exceptions exist. Naturally, those with the most skin in the game take economic competition into the arena of politics, which adds certain wrinkles to the underlying political economy of the nation. And that's a pretty important set of wrinkles given that the Hermesiatics are institutionalizing into a modern state!

Let me show you what I've been thinking about whilst interpreting Hermesiatic dealings. While we're there, we might dabble in some light stonk trading, yes? Mmm, stonks. Smash!

THE FOUR HOUSES

EVLOGÍA

KLEIDÍ

SFYRÍ

SFÍNGA

There are four main trading houses in the Hermesiatics: The Blessing, The Key, The Hammer, and The Sphinx. Though all of them maintain wide and diverse interests across Olympia, each has its own niche: The Blessing focuses on religious iconography and the development of new theological products; The Key is primarily a financial house with interests in equities, real estate, and basic commodities; The Hammer specializes in manufacturing, with industrial holdings in nearly every Olympian state; and The Sphinx is a trading house that charges excellent mark-ups for bringing exotic goods across Olympian oceans. These four houses have dominated Hermesiatic, and indeed Olympian, markets for generations.

But here a problem arises: as the Hermesiatics transform from a loose confederation of market-oriented city-states to a nation-state proper, new governmental institutions are being developed rapidly. These institutions threaten the status quo that have made the Four Great Hermesiatic Houses so rich for so long, and so it stands to reason that the Houses aim to shape the institutions as they are being developed. But, they do not enjoy their usual sense of agency in these matters, as (for once) they cannot specialize their way into an isolated niche; they must engage with one another, be it for purposes of cooperation or of conflict, in the name of influencing political outcomes.

Naturally, the Houses deal with one another all the time—after all, they often sell the same good in the same market. But is that kind of interaction the same as these new institutionalized ones? In what senses might they be similar, and in what senses might they differ? Is all competition the same?

These are hard questions, and I'd just as soon not get too far into the weeds. And yet, something there is that demands an answer....

PART 1 | AT WHAT PRICE?!

Like I said, the houses often find themselves in economic competition with one another. Why, just the other day I read that The Key and The Hammer were dueling it out in the market for lyres. Don't laugh—lyres are a pretty big deal here in the Hermesiatics, and they fetch a pretty penny in Phoebustan!

But here's the problem: as the houses make more and more lyres, the price people are willing to pay for lyres goes down. So on the one hand, you make more money by making more lyres. But on the other hand, you make less money per lyre as you make more and more! And one other teeny tiny detail: the other house is making lyres, too, which means they could make more money while driving your price down!

We therefore enter into the following situation:

- there are two houses: K (for Key) and H (for Hammer);
- each house h chooses some quantity of lyres to make, where this is encoded $q_K \in \{0, 1, 2\}$ and $q_H \in \{0, 1, 2, 3\}$;
- the cost of producing q lyres is μq, so that $\mu > 0$ encodes the marginal cost of production; and
- if the two houses produce $Q = q_K + q_H$ lyres in total, then buyers will pay a price of $5 - Q$ per lyre.

House utilities are determined by total profits:

$$u_K(q_K, q_H) = (5 - q_K - q_H)\, q_K - \mu q_K, \text{ and}$$
$$u_H(q_K, q_H) = (5 - q_K - q_H)\, q_H - \mu q_H.$$

1. What is the set of all strategy profiles for this game?
2. What is the price buyers will pay if The Key makes one lyre and The Hammer makes two?
3. What is The Hammer's profit if The Key makes two lyres, the Hammer makes one, and the marginal cost of production is one half?
4. Present the game in matrix form for general μ. You need only fill in the "bottom" of the matrix (plus the diagonal), as the game is symmetric.
5. Suppose $\mu = 1$; what is the set of Pareto-optimal strategy profiles?
6. Is your answer to (5) robust to any μ?
7. For a given player i, we say strategy a_i' *strictly dominates* strategy a_i'' if the following holds: for all other players' strategies a_{-i}, we have $u_i\left(a_i', a_{-i}\right) > u_i\left(a_i'', a_{-i}\right)$. In other words, one strategy strictly dominates another iff the first is always strictly better than the second, no matter what other players do. Again supposing $\mu = 1$, does either house have a strictly-dominated strategy? If so, what dominates it?

necessary for **PASS**: get 3

sufficient for one **ALMA**: get 7

sufficient for another **ALMA**: suppose you studied a new game constructed by removing any strictly-dominated strategies from the old game. would you be able to remove any more strategies? that is, if we iteratively eliminate strictly-dominated strategies, what game do we wind up with? what is the substantive interpretation of the game's IESDS residue?

PART 2 | IN THE LOBBY

Meanwhile, I've heard some reports of a nasty battle brewing between the traders at The Blessing and The Sphinx. But this time it's not about product-it's about whose preferred agent fills the new job of Associate Assistant to the Deputy Undersecretary of Import/Export/Endoport/Exoport/Intraport/Interport/Miscellport/Antiport/Proport/Quasiport/Expliciport Affairs, Class VII. It's an important job, as you must be able to tell. Naturally, The Blessing is pushing for a virtuous clerk with an eye toward keeping corruption down, whereas The Sphinx wants somebody a little more (shall we say) worldly at the post.

At any rate, the two houses are engaged in a campaign against one another to try to fill the seat. It works as follows:

- we have two houses: B (for Blessing) and S (for Sphinx);
- each house h chooses lobbying effort $e_h \in [0, 1]$, where this tells us the proportion of total resources devoted to the lobbying campaign;
- given the two effort levels, the probability that The Blessing's clerk is given the job is

$$\pi_B(e_B, e_S) = \begin{cases} \frac{1}{2}, & e_B + e_H = 0 \\ \frac{e_B}{e_B + e_H}, & e_B + e_H > 0 \end{cases};$$

- The Sphinx's probability of winning is analogous, and indeed is simply $1 - \pi_B(e_B, e_S)$;
- the cost of expending effort e_h is κe_h, so that $\kappa > 0$ encodes the marginal cost of lobbying effort; and
- filling the job with your preferred candidate gets you V happiness points, and losing gets you 0.

We arrive at each house's expected utility of lobbying,

$$U_h(e_B, e_S) = \pi_h(e_B, e_S)\, V - \kappa e_h.$$

Now:

1. Suppose The Blessing works at 40% intensity and The Sphinx works at 60% intensity. What are the respective probabilities of victory for the two houses?
2. What is The Sphinx's expected utility of lobbing when The Blessing works at 50% intensity, The Sphinx works at 20% intensity, and $\kappa = 1$?
3. Suppose the two houses can only choose intensity levels from the set $\{0, 1/3, 1/2, 2/3, 1\}$. Present the game in matrix form for general κ. You need only fill in the "bottom" of the matrix (plus the diagonal), as the game is symmetric.
4. Suppose the houses could instead just divvy up the V happiness points between themselves, with The Blessing getting v and The Sphinx getting $V - v$. Is it the case that for all strategy profiles (e_B, e_S), there exists a $v \in [0, V]$ such that $v \geqslant U_B(e_B, e_S)$ and $V - v \geqslant U_S(e_B, e_S)$?

necessary for **PASS**: get 1

sufficient for one **ALMA**: get 3

sufficient for another **ALMA**: consider the following version of the contest:

$$\pi_B(e_B, e_S) = \frac{\lambda e_B^\gamma}{\lambda e_B^\gamma + e_S^\gamma}$$

where $\gamma \in [0, 1]$ is called *discrimination* and $\lambda > 0$ is called *relative technology*. Study this a bit and explain why those parameters are so named. What happens at low values of these parameters? At high values? [*Hint:* you might be tempted to write "bang for the buck."]