GENESEO

PROBLEM SET 08

THREE HEADS? NO PROBLEM

Χαῖρε, friend. I just saw in the Olympian Φορές that a weapons inspector has been sent to Hadestum. Officials from Herania, under consultation with allies Artemissia and n'Zeusk, have given reason to believe that the Hadestans are arming themselves in preparation to attack Herania. The most likely motive is a desire to re-acquire territories lost in the Isthmus of Cerberus splitting Lakes Hippolyta and Heracles to the Sea of Myrtles; this area has become more desirable of late, particularly near the planned city of New Rodi. Globalists at the Olympian Prytaneion have commissioned a special inspector, Krona, though they are not yet sure how the incentives of the position play out.

It doesn't help matters that Herania and Hadestum have a complicated history on matters territorial; this wouldn't be the first time tempers have flared for control of the Isthmus of Cerberus, and if you'll forgive my cynicism—eat your heart out, Diogenes!— it likely won't be the last.

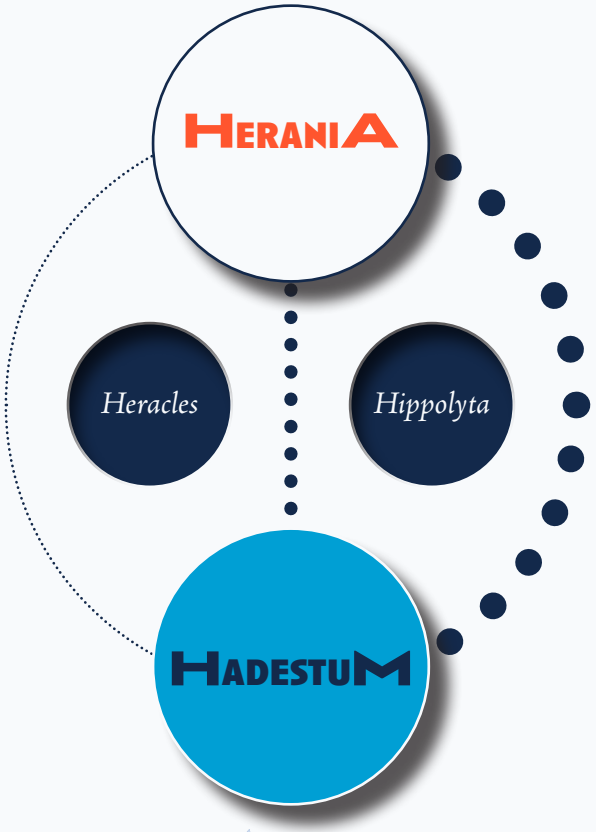
I wish we were ruminating under better circumstances, but alas, it was not meant to be. But then again, don't we keep our swords sharp that we may think clearly when that's what's needed most?

SOCRATES

HOWL FROM HELL

The Isthmus of Cerberus has long been a source of international tensions; it's changed hands between Herania and Hadestum several times, and those of us with long memories have gotten the sneaking feeling that we're due for a skirmish. The question is which route the Hadestans would take to try to re-claim the Isthmus: they could go west, launching an initiative from Minos, circling Lake Heracles, and on through Wetumpka; they could go east, launching an initiative from Thanatos, braving the foothills of the Letosian Mountains, raiding through Mycenae and eventually arriving at New Rodi; or, they could go down the middle, launching an inititive from Tartarus and fighting through the bottleneck between the two lakes.

As you may have gathered, these routes are not made equal: the western route is definitely the easiest, and the eastern route is the most difficult. Nevertheless, previous conflicts suggest that the attacking state occasionally takes the hard route, which has been difficult for military theorists to make any sense of. Why in Olympia would an attacker take the eastern route, given its level of difficulty? It makes little sense, but maybe there's more going on underneath the hood.



Naturally, these little conflicts always bring up some degree of brinksmanship between the old rivals, and over the years I've come to understand their bizarre rituals a little bit. Let's just hope Krona can nip this all in the bud with a successful inspection! She seems pretty wise, but is she smart?

PART 1 | THE INSPECTION

Krona's got a tough job on her hands! For starters, the Hadestans can be a tough bunch to work with; they're always surly and they're super secretive and *oh my gods would you stop talking about death all the time we get it*. But also, inspecting Hadestum takes time and effort, and she'd like to keep that to an abosolute minimum while also bringing out the best possible result. How angry would you be if you spent all that time traipsing around the Infernal Regions only to find out the bastard have been behaving all along?! Meanwhile, the Hadestans have to figure out whether the incentives the Globalists are offering outweigh the risks of going undefended in the future; after all, might makes right today, but it makes even righter tomorrow.

NOTATION	CONCEPT	RANGE
k	Krona's inspection costs	>0
g	Krona's penalty for a crisis	$>k$
m	Krona's reward for no building	>0
c	Hadestum's reward for compliance	>0
f	Hadestum's strategic cost of weakness	>0

I think the interaction is actually pretty basic:

HADESTUM \ KRONA	INSPECT	DON'T
BUILD WEAPONS	0 $-k$	c $-g$
DON'T	$c-f$ $m-k$	$c-f$ m

oh hades yes:

1. Suppose Hadestum's foregone security cost strictly outweighs her compliance benefit. Identify all Nash equilbiria (be they in pure or mixed strategies).
2. Now suppose Hadestum's foregone security cost is strictly outweighed by her compliance benefit. Identify all Nash equilbria.
3. Do your answers differ? Why or why not?
4. For each of the equilbria you identify for Questions 1 and 2, what is the probabily of observing each strategy profile in equilibrium?
5. For all mixed equilibria, determine how the relevant mixing probabilities depend on the exogenous terms. What is increasing in what? What is decreasing in what?

necessary for **PASS**: get 3

sufficient for one **ALMA**: get 5

sufficient for another **ALMA**: now suppose the foregone security cost is eqactly equal to the compliance benefit. identify *all* Nash equilibria.

PART 2 | HOW BAD IS THE BRINK?

The last time this happened, Herania and Hadestum wound up taking the crisis right up to the breaking point! It occurred to me that the more things change, the more they stay the same: all the two states are interested in are winning and losing relative to one another. What *does* change, however, is how awful it is to let things escalate to an actual conflict: weapons are ever stronger, and armies are ever larger, and bombs are ever boomier, and...

It pains me to say it, but we need a general way to think about what to expect as a function of how awful a war between these two states would be. Let us suppose that the cost of a conflict is a simple term $\lambda_E > 0$ for Herania and $\lambda_A > 0$ for Hadestum. I only hope that we need not interpret these parameters with too much care!

HADESTUM \ HERANIA	BACK DOWN	MARCH FORWARD
BACK DOWN	1 1	0 2
MARCH FORWARD	2 0	$-\lambda_A$ $-\lambda_E$

Lambdas make me want lamb.

1. Which *pure* strategy profiles are Pareto optimal?
2. Which *mixed* strategy profiles are Pareto optimal?
3. Identify all pure-strategy Nash equilibria.
4. Identify all mixed-strategy Nash equilibria.
5. For each Nash equilibrium, what is the probability each of the pure strategy profiles occurs?
6. For each Nash equilibrium, what is each player's expected utility for the game in that equilibrium?
7. Put all pure strategy profiles and all equilibrium utilities into a utility imputation space. Show how the mixed equilibrium's location depends on the costs of conflict.

necessary for **PASS**: get 4

sufficient for one **ALMA**: get 7

sufficient for another **ALMA**: for funsies, suppose one of the states has zero costs of conflict. identify *all* Nash equilibria.

PART 3 | ITS UGLY HEAD

Finally, let's talk tactics. As I mentioned before: not all roads to New Rodi are the same, and this will likely influence how the Hadestans attack (if they do). Naturally, the fact that the Hadestans' attack is influenced itself influences the Heranian decisions with respect to defense, which makes this all quite messy.

The easiest route to New Rodi goes to the east; the second easiest is in the center; and the hardest is to the west. The Hadestans want to attack a place where the Heranians are not defended, but they want to do so as easily as possible. Meanwhile, the Heranians want to set up a defense where the Hadestans have attacked; they don't care much otherwise.

To encode this, I've set the happiness points for the best outcome at 1 and the worst outcome at 0. The Hadestans would love to go to the west; going center is c_C happiness points worse, and going east is c_E happiness points worse (where we have $0 < c_C < c_E < 1$).

HADESTUM \ HERANIA	EAST	CENTER	WEST
EAST	0 1	1 0	1 0
CENTER	$1 - c_C$ 0	0 1	$1 - c_C$ 0
WEST	$1 - c_E$ 0	$1 - c_E$ 0	0 1

I can't tell if this one is easy or hard.

1. Identify all pure-strategy Nash equilibria.
2. Does there exist an equilibrium where one state plays a pure strategy and the other mixes?
3. Does there exist an equilibrium where both states mix over two locations?
4. Does there exist an equilibrium where both states mix over all three locations?
5. How do the relevant mixing probability depend on the relative costs of attacking the three different locations?
6. For each equilibrium, determine the expected utility for each state in playing the game. Given these expected utilities, does there exist a peaceful division of one happiness point that Pareto dominates the equilibrium outcome?

necessary for **PASS**: get 3

sufficient for one **ALMA**: get 6

sufficient for another **ALMA**: let's call the number of locations each state mixes over a *kind of strategy*. thus, there is a kind of strategy where both states go center for sure. there's another where Herania mixes over west and east and Hadestum mixes over all three. there's another kind where both states mix over all three locations. how many kinds of strategy are there in total? how many of those kinds of strategy support a Nash equilibrium?