PROBLEM SET 10

ULTIMA RATIO

Χαίρε, friend. I've been so depressed watching the news that I barely have the strength to check in with you. Alas, the technocrats didn't say "hey Socrates, why don't you just work on solving society on days where you're feeling inspired?" *Technocrats do not know what inspiration feels like.* The word is ill-posed to them, like "joy" or "intelligence."

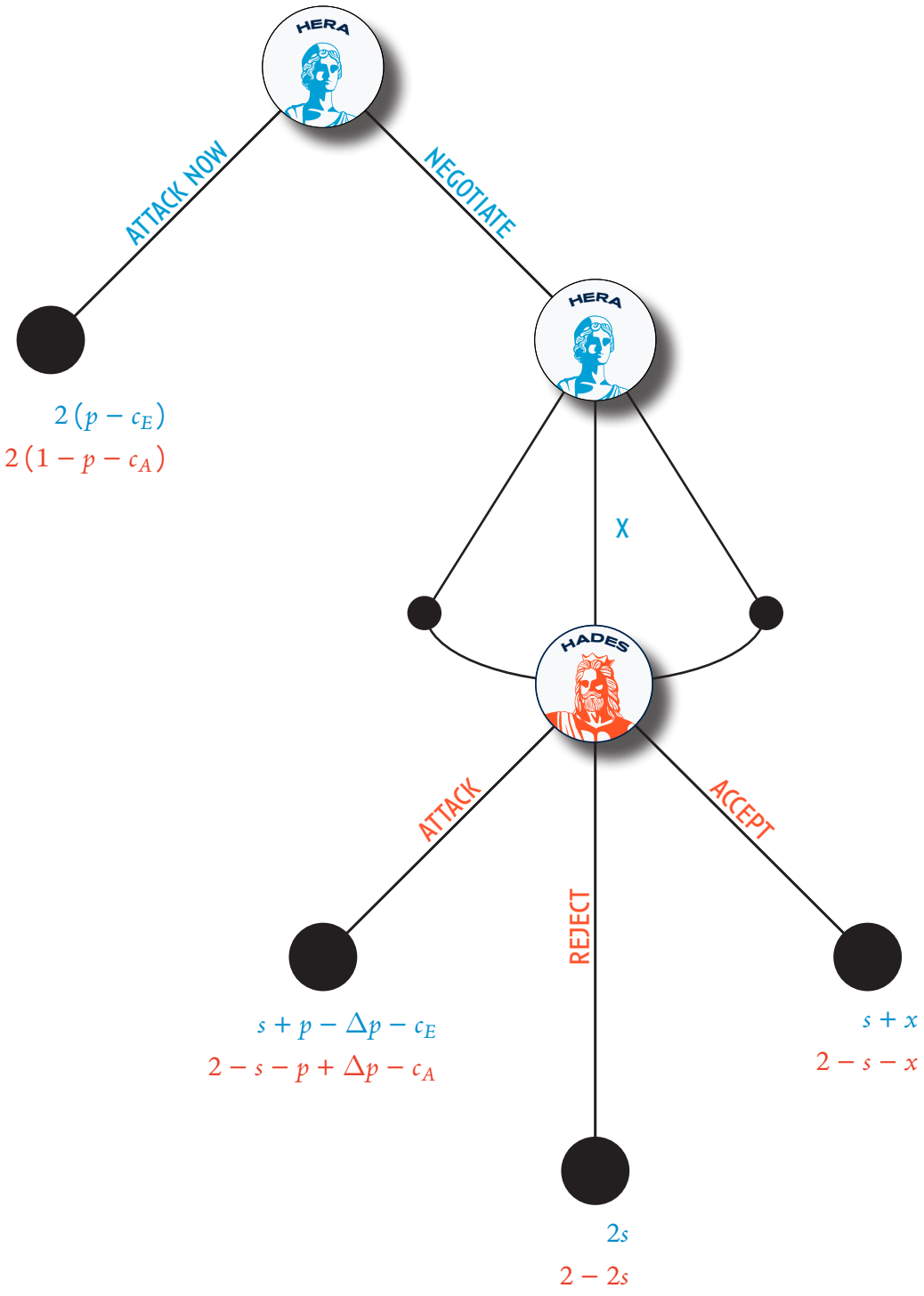
Anywho, the reason I'm depressed is that it seems to me that we could in fact be on the brink of war. Now that the Hadestans have chosen an angle of attack as they march on New Rodi, it seems that we've little choice but to hope that the two states can work out an agreement. But, it seems to me that any deal the states sign to make peace will be, in a word, fragile: after all, Hadestum continues to arm itself and continues to grow stronger. The Heranians have the upper hand now, but will that continue into the future?

Meanwhile, it seems to me that the Hadestans are getting stronger at extreme domestic cost. Do you think they'll be able to keep that going, or will the people wind up revolting against them? After all, it's not like they'll get to enjoy the spoils of war!

Tragedy, tragedy everywhere. This is when we have the deepest obligations to think hard and think thoroughly.

SOCRATES

PART 1 | MAKE YOUR MOVE



Concept	Symbol	Constraint
Initial probability Heranians win	p	$p \in [0, 1]$
Heranians' cost of war	c_E	$c_E > 0$
Hadestans' cost of war	c_A	$c_A > 0$
Heranians' status quo happiness share	s	$s \in [0, 1]$
Heranians' renegotiation offer	x	$x \in [0, 1]$
Increase in Hadestans' power	Δp	$\Delta p \in [0, p]$

Here's how I'm seeing this go down:

At the beginning of time:

1. there's a status quo division of land that gives the Heranians s happiness points and the Hadestans $1 - s$ happiness points.
2. the Heranians and Hadestans have forces such that the Heranians would win a fight with probability $0 \leqslant p \leqslant 1$. Naturally, both states would have to pay war costs for that.

So early on, the Heranians have a choice: they can attack the Hadestans while they're still strong and get their initial expected utility of war for two time periods, or they can try to re-negotiate the status quo by choosing some $0 \leqslant x \leqslant 1$.

But by the time we've arrived at this moment in time, the Hadestans are stronger! How much stronger? Well, their probability of victory goes from $1 - p$ to $1 - p + \Delta p$, where $0 \leqslant \Delta p \leqslant p$ encodes their newfound power by telling us how much more likely they are to win. Naturally, the Heranians lose the same amount from their probability of victory.

The Hadestans can do one of three things: they can attack with their newfound power, getting one round of status quo peace utilities and one round of the new expected utility of war; or, they can reject the deal and accept two rounds of the status quo peace utilities; or, they can accept the deal and get one round of status quo and one round of renegotiated terms.

Let's get this right, please:

1. Consider a simultaneous-move version of the game starting from the Heranians' decision node regarding the offer. (That is, ignore the first choice for now.) Identify all pure-strategy Nash equilibria.
2. Does war ever occur in any of these equilibria?
3. Is it the case that these equilibria generate Pareto-optimal outcomes?
4. Now suppose the game began from the Heranians' decision of what to offer but includes the time structure as drawn. Identify all subgame perfect equilibria (in pure strategies).
5. Does war ever occur in any of these equilibria?
6. Is it the case that these equilibria generate Pareto-optimal outcomes?
7. Now suppose we're playing the full game with the time structure. Identify all subgame perfect equilibria (in pure strategies).
8. How often does war occur in the equilibria you just unearthed?
9. Is it the case that these equilibria generate Pareto-optimal outcomes?

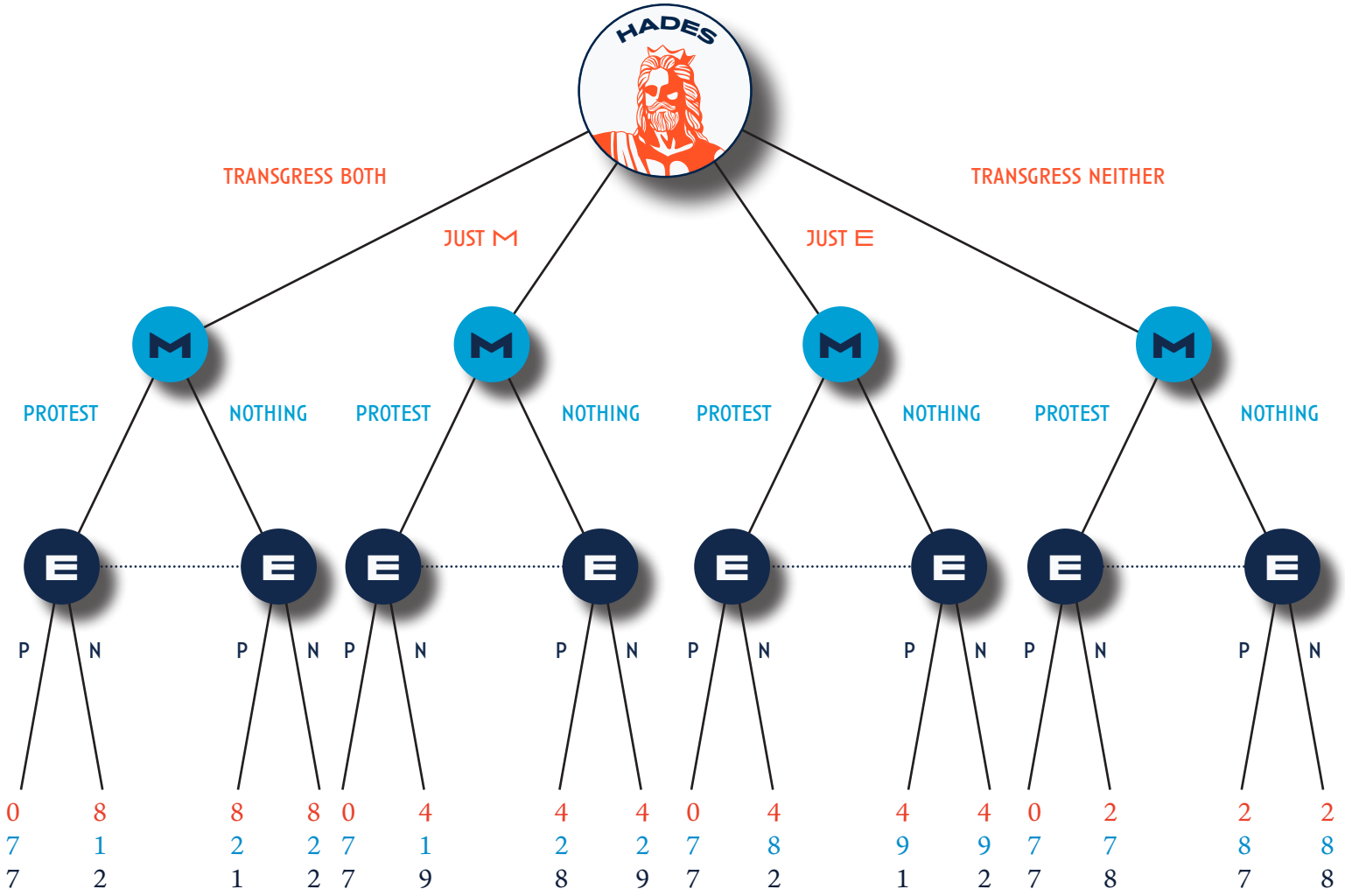
necessary for **PASS**: get 6

sufficient for one **ALMA**: get 9

sufficient for another **ALMA**: who wrote that the reason for the Peloponnesian War was the rise in Athenian power and the fear this caused in Sparta? In 750–1000 words, discuss the relevance of this model to that concept.

PART 2 | PITCHFORKS!

Meanwhile, this increase in Hadestan power didn't come from *nowhere*: they're forcing the various factions in their society to work to the bone to increase military-industrial output! And what's worse, they're playing the factions against one another, to boot! Broadly speaking, there are two rank-and-file factions in Hadestum: the Macarians and the Erinyesians. And any Hadestan can tell a Macarian from an Erinyesian based on their appearance and accent, so it's possible to target one faction or the other for the purposes of nefarious mobilization!



The big problem here is that the Macarians and Erinysians need to coordinate with one another if they want to stop the elites from transgressing their labor rights, but (1) they can't see one another well enough to coordinate on that, and (2) the elites can manage to buy one faction off against the other. The game is as drawn above.

Throw the bastards out!

1. Which outcomes are Pareto optimal?
2. How many subgames does this game have?
3. Identify all subgame perfect equilibria.
4. Evaluate: a profile is subgame perfect if and only if it is optimal. If so, why? If not, can you weaken the claim?
5. Evaluate: there exists a subgame perfect equilibrium where no rights are transgressed. If so, why? If not, why not?
6. Evaluate: there exists a subgame perfect equilibrium where one faction protests and the other does nothing. If so, why? If not, why not?
7. Evaluate: there exists a subgame perfect equilibrium where both factions' rights are transgressed. If so, why? If not, why not?

necessary for **PASS**: get 5

sufficient for one **ALMA**: get 7

sufficient for another **ALMA**: identify all subgame perfect equilibria in pure or mixed strategies.