PROBLEM SET 12

THE MORE YOU KNOW?

Socrates

Χαίρε, friend. Can you believe how long we've been walking this path together? It boggles the mind.

I saw in the news that Phœbustan is due for a public referendum on bow rights: their constitution offers blanket protections on bows, as bows have long been an important part of Apollonian culture. Nevertheless, many Sunheads have been trying to walk those protections back after a spate of bow attacks. Needless to say, the matter is quite controversial.

But you know what I wish? I wish that the relevant parties could come together and have a real conversation about where they're coming from. It just seems like the absence of such conversation makes it hard to achieve good outcomes. Maybe you'd like to ruminate on the topic with me a bit. We begin.

PART 1 | SWING!

Remarkably, I can say with certainty that all Sunheads agree that they want what is best for society; theirs is a happy culture, and they've quite a bit of faith in their nation, state, and so on. The difficulty on the topic of bow rights is that not everybody knows what is best, at least not perfectly so: a group of elites has run some studies to learn about the best policy, but the public at large doesn't know for sure, though they've got a guess.

ELITES

PRO-BOW	ABSTAIN	ANTI-BOW
+1, +1	+1, +1	±0, ±0
+1, +1	±0, ±0	-1, -1
±0, ±0	-1, -1	-1, -1

α

PRO-BOW	ABSTAIN	ANTI-BOW
-1, -1	-1, -1	±0, ±0
-1, -1	±0, ±0	+1, +1
±0, ±0	+1, +1	+1, +1

$1 - \alpha$

PRO-BOW

ABSTAIN

ANTI-BOW

MASSES

In other words, the Elites can condition their support for or against bows on what is best for society, whereas the Masses cannot do so: instead, they can only do what they think is best in expectation. Formally, they think that a Pro-Bow stance is best for society with probability α, and they think an Anti-Bow stance is best with probability $1 - \alpha$. Remarkably, both the Elites and the Masses have the ability to do nothing.

Are you Pro-Bow? No? Pro...Bow...No.....?

1. In words, characterize the primary necessary condition for any outcome to be Pareto optimal. (This should not take you more than one sentence.)
2. Is this a bargaining problem? Why or why not?
3. What bucket contains the Elites' strategy? What bucket contains the Masses' strategy?
4. How many strategy profiles does the game have?
5. Suppose the Elites decide to stay out of the policy arena, choosing instead to always Abstain. What is the Masses' expected utility for each of their available strategies?
6. What is the Masses' best response if the Elites always Abstain? (This might depend on certain factors....)
7. Now suppose the Elites choose to always vote for what's best for society. What is the Masses' expected utility for each of their available strategies?
8. What is the Masses' best response if the Elites always vote for what is best for Society?
9. What is the best response for each type of Elite in the event that the Masses Abstain?

necessary for **PASS**: get 6

sufficient for one **ALMA**: get 9

Given all this ambiguity in the prevailing policy environment, the market forces in the bow industry are definitely in flux! The biggest bowmaking firm, Chrysus, is thinking about trying to take over the second biggest firm, Arc. The problem is that the good people at Arc know more about their own company than the would-be hostile takers-over over at Chrysus, even though the latter could do a far better job running the company.

Suppose Arc is worth n Phoebes, where n is an integer between 0 and 100. Suppose Arc knows how many Phoebes they're worth, call it x, but that Chrysus does not; instead, Chrysus thinks each of the 101 possible valuations is equally likely.

The game is simple: Arc chooses a number X between 0 and 100, where they accept all offers less than or equal to X and reject all offers strictly greater. Simultaneously, Chrysus chooses a bid Y. If X is no greater than Y, then the offer is accepted: Arc gets Y happiness points for making the sale, and Chrysus gets $1.5x - Y$ happiness points for their new purchase. If X is strictly greater than Y, then Arc gets x happiness points for keeping the business going and Chrysus gets zero.

Tricky tricky

1. Suppose Chrysus actually knew the value of Arc. Draw a stylized matrix depicting the game in strategic form.
2. Again supposing for now that Chrysus knows the value of Arc, characterize all pure-strategy Nash equilivbria of the complete-information version of the game.
3. Characterize the set of all optimal outcomes in the complete-information version.
4. Now suppose Chrysus doesn't know the value of Arc (as in the game). What is Chrysus's expected utility of offering $X = 0$ as a bid? 50? 100?
5. Suppose $x = 50$. What is Arc's best response to a bid of 0? 50? 100?
6. Repeat Question 5 for when $x = 0$.
7. Repeat Question 5 for when $x = 100$.

necessary for **PASS**: get 5

sufficient for one **ALMA**: get 7

sufficient for another **ALMA**: why do you think that the mechanism at work here is sometimes called "adverse selection?"